Effects of Flow Unsteadiness on Hypersonic Wind-Tunnel Spectroscopic Diagnostics

Thomas M. Weeks*
Air Force Flight Dynamics Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio

An analysis is presented of nonlinear effects of time varying flow properties on interpretation of spectroscopic measurements. Both direct emission as well as electron beam techniques are considered. A method, based on this analysis, is evolved whereby both average and mean square fluctuating temperatures can be obtained using conventional instrumentation. The method is applied to several selected experimental cases cited in the literature, including recent arc-heated hypersonic wind tunnel electron beam measurements. The theory is extended to include species number density fluctuations when temperature fluctuations are also present.

Nomenclature

A_n^m = spontaneous transition probability
c = speed of light
E_m = electron energy level
$G(v_i)$ = vibrational term value ($N_2 \times 1\Sigma$)
g_m = statistical weight
h = Planck's constant
I = line or band intensity
k = Boltzmann's constant
K' = rotational quantum number
$q(v',v'')$ = Franck-Condon factor
R = line or band intensity ratio
T = static temperature
ν = frequency
θ_r = characteristic rotational temperature

Subscripts

e = excitation
0 = reference conditions
r = rotational
v = vibrational

I. Introduction

Spectroscopic diagnostic techniques are commonplace in hypersonic wind tunnel investigations. Until the present time, however, little attention has been given to the possible effects of flow unsteadiness on interpretation of measured line intensities. The usual assumptions are either that the flow is steady and, therefore, that line intensities recorded with conventional time averaging equipment are indeed constant or that while the flow may be unsteady the average value of the line intensities yield directly the average temperature. Neither assumption is valid in general and a more careful approach is required. This involves writing the governing relation between observed line intensity and corresponding thermodynamic property, expanding it in a Taylor's series and taking a time average. This appears to be the most direct approach although several others leading to essentially similar results have appeared.[1-3]

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* Aerospace Engineer, Flight Mechanics Division. Associate Fellow AIAA.

II. Analytical Approach

The ratio R of two spectral line intensities can often be expressed as function of temperature in the form

$$R(T) = I/I_0 \tag{1}$$

Letting $T = \bar{T} + T'$ a Taylor's series truncated to second order can be written

$$R(T) = R(\bar{T}) + T'\left(\frac{dR}{dT}\right)_{T=\bar{T}} + \frac{T'^2}{2}\left(\frac{d^2R}{dT^2}\right)_{T=\bar{T}} \tag{2}$$

The time average of Eq. (2) is ($^-$ and $\langle\rangle$ denote time average)

$$\bar{R}(T) = R(\bar{T}) + \frac{\langle T'^2\rangle}{2}\left(\frac{d^2R}{dT^2}\right)_{T=\bar{T}} \tag{3}$$

In the absence of flow fluctuations $\overline{R(T)} = R(\bar{T})$. This is essentially the assumption often made even when flow fluctuations are present.

The procedure in all cases treated here is to evaluate $(d^2R/dT^2)_{T=\bar{T}}$ for the particular governing relationship indicated by Eq. (1). By providing sufficient intensity measurements both $\bar{T}$ and $\langle T'^2\rangle$ can be determined.

III. Direct Emission Analysis

The ratio of relative intensities of two spectral lines of a self-luminous gas separated by excitation energy ΔE_{10} can be expressed as

$$R(T) = I_1/I_0 = \exp - \Delta E_{10}/kT_e \tag{4}$$

where T_e is the excitation temperature. We then have

$$\left(\frac{d^2R}{dT^2}\right)_{T=T_e} = \frac{1}{\bar{T}_e{}^2}\left[\frac{\Delta E_{10}}{k\bar{T}_e}\left(\frac{\Delta E_{10}}{k\bar{T}_e} - 2\right)\right]\exp(-\Delta E_{10}/k\bar{T}_e) \tag{5}$$

Notice that when $\Delta E_{10}/kT_e = 2$ there is no effect of temperature fluctuation on measured line intensity. This result was obtained and discussed in Ref. 2 and arises due to the existence of an inflection in Eq. (4) at $\Delta E_{10}/kT_e = 2$. If lines satisfying this criteria are present in the spectrum then $\bar{R}(T_e) = R(\bar{T}_e)$ for such lines and $\bar{T}_e$ can be found directly. This condition is to be avoided in the present case as we are seeking both $\langle T_e'^2\rangle$ as well as $\bar{T}_e$.

At this point an examination of the relation of $\overline{R(T_e)}$ to measured line intensity ratio must be made. Conventional recording equipment (e.g., spectrographic plate, photomultiplier plus strip chart, etc.) yields $\bar{I}$ and $\bar{I}_0$.

Introducing $I = \bar{I} + I'$ we may write

$$R(T_e) = (\bar{I}_1 + I_1')/(\bar{I}_0 + I_0') \tag{6}$$

Carrying out the division, dropping terms of order higher than second in primed quantities and taking the time average yields

$$\overline{R(T)} = \frac{\bar{I}_1}{\bar{I}_0}\left(1 - \frac{\langle I_1'I_0'\rangle}{\bar{I}_1\bar{I}_0} + \frac{\langle I_0'^2\rangle}{\bar{I}_0^2}\right) \tag{7}$$

With equipment capable of obtaining time resolved measurements of I_1 and I_0 independently the quantities in Eq. (7) could be obtained. With only conventional equipment one looks for a simplification of the form

$$\overline{R(T)} \approx \bar{I}_1/\bar{I}_0 \tag{8}$$

This requires that the quantity

$$J_{10} = \frac{\langle I_0'^2\rangle}{\bar{I}_0^2} - \frac{\langle I_1'I_0'\rangle}{\bar{I}_1\bar{I}_0} \ll 1 \tag{9}$$

which in turn depends on the experimental realization of one of two situations. Either 1) $I_1/\bar{I}_1$ and $I_0'/\bar{I}_0$ must be small, or 2) $\langle I_1'^2\rangle/\bar{I}_1 \sim \langle I_0'^2\rangle/\bar{I}_0^2$ and $\langle I_1'I_0'\rangle/\bar{I}_1\bar{I}_0 > 0$ (positive correlation among selected line intensity fluctuations). The first condition has already been assumed to maintain second order in Eq. (3). If in a particular situation the second condition is satisfied then $J_{10} \to 0$ independent of fluctuation levels thus improving the degree of approximation Eq. (8). Adopting the condition given in Eq. (9) permits Eq. (3) to be written as

$$\frac{\bar{I}_1}{\bar{I}_0} = \exp\frac{-\Delta E_{10}}{k\bar{T}_e}\left\{1 + \frac{\langle T_e'^2\rangle}{2\bar{T}_e^2}\left[\frac{\Delta E_{10}}{k\bar{T}_e}\left(\frac{\Delta E_{10}}{k\bar{T}_e} - 2\right)\right]\right\} \tag{10}$$

One now needs only to select a third line of the emission spectrum with intensity I_2 and write

$$\frac{\bar{I}_2}{\bar{I}_0} = \exp\frac{-\Delta E_{20}}{k\bar{T}_e}\left\{1 + \frac{\langle T_e'^2\rangle}{2\bar{T}_e^2}\left[\frac{\Delta E_{20}}{k\bar{T}_e}\left(\frac{\Delta E_{20}}{k\bar{T}_e} - 2\right)\right]\right\} \tag{11}$$

where we must insure that $\Delta E_{10} \neq \Delta E_{20}$. Thus using the measured quantities $\bar{I}_0$, $\bar{I}_1$, $\bar{I}_2$ taken for example from a spectrographic plate one can solve Eqs. (10) and (11) simultaneously for $\langle T_e'^2\rangle$ and $\bar{T}_e$.

IV. Electron Beam Analysis

Rotational Temperature

We turn our attention to the essentially similar treatment of electron beam determination of rotational temperature. For temperatures below 800°K the ratio of two line intensities in the 1st negative system of nitrogen are related to the rotational temperature T_r by[4]

$$\frac{I_{K'}}{I_{K_0'}} = \frac{GK'}{G_0K_0'}\left\{\exp\frac{-[K'(K'+1) - K_0'(K_0'+1)]\theta r}{T_r}\right\} \tag{12}$$

Introducing $I = I_K'/GK'$, $X = K'(K'+1) - K_0'(K_0'+1)$ Eq. (1) becomes

$$R(T_r) = I/I_0 = \exp - X\theta_r/T_r \tag{13}$$

Here we find

$$\overline{R(T_r)} = R(\bar{T}_r)\left\{1 + \frac{\langle T_r'^2\rangle}{\bar{T}_r^2}\left[\frac{\theta_r X}{2\bar{T}_r}\left(\frac{\theta_r X}{\bar{T}_r} - 2\right)\right]\right\} \tag{14}$$

The left-hand side of Eq. (14) may be treated as under the previous section (i.e., $\bar{R} \approx \bar{I}/\bar{I}_0$).

The procedure in the steady flow case for obtaining T_r from measured I/I_0 is to fit a straight line to a plot of log intensity ratio vs K' $(K' + 1)$. The slope of such a straight line yields θ_r/T_r. In the presence of fluctuations the full Eq. (14) can be similarly evaluated. Taking the log of Eq. (14) we have

$$\ln\frac{\bar{I}}{\bar{I}_0} = X\frac{\theta_r}{\bar{T}_r} + \ln\left\{1 + \frac{\langle T_r'^2\rangle}{\bar{T}_r^2}\left[\frac{\theta_r X}{2\bar{T}_r}\left(\frac{\theta_r X}{\bar{T}_r} - 2\right)\right]\right\} \tag{15}$$

Then for

$$\left|\frac{\langle T_r'^2\rangle}{\bar{T}_r^2}\left[\frac{\theta_r X}{2\bar{T}_r}\left(\frac{\theta_r X}{\bar{T}_r} - 2\right)\right]\right| \ll 1$$

we may invoke the approximation $\ln(1+\epsilon) \approx \epsilon$ yielding

$$\ln\frac{\bar{I}}{\bar{I}_0} = -\frac{\theta_r}{\bar{T}_r}\left(1 + \frac{\langle T'^2\rangle}{\bar{T}_r^2}\right)X + \frac{1}{2}\frac{\langle T_r'^2\rangle}{\bar{T}_r^2}\left(\frac{\theta_r}{\bar{T}_r}\right)^2 X^2 \tag{16}$$

Thus instead of the linear Boltzmann plot we are dealing according to Eq. (16) with a second-order relationship between log intensity ratio and X of the form

$$\ln(\bar{I}/\bar{I}_0) = a_1X + a_2X^2$$

where

$$a_1 = (-\theta_r/\bar{T}_r)(1 + \langle T_r'^2\rangle/\bar{T}_r^2)$$
$$a_2 = \tfrac{1}{2}(\theta_r/\bar{T}_r)^2(\langle T_r'^2\rangle/\bar{T}_r^2)$$

or more directly

$$\bar{T}_r = 2\theta_r/[(a_1^2 - 8a_2)^{1/2} - a_1] \tag{17}$$

$$\langle T_r'^2\rangle/\bar{T}_r^2 = 1 + a_1\bar{T}_r/\theta_r \tag{18}$$

Notice that $a_2 \geq 0$ while $a_1 < 0$. Thus the Boltzmann plot with negative slope has positive curvature for all finite but small $\langle T_r'^2\rangle/\bar{T}_r^2$. A parabolic shape similar to that noted in Eq. 15 has been noted and often attributed to non-Boltzmann equilibrium effects in the flowing system. Similar effects are noted in static calibration tests and might be due to heating of the gas by the beam while a scan is in progress. In flowing gases the presence of turbulence or unsteadiness apparently contributes additional curvature effects.

Vibrational Temperature

To illustrate the vibrational temperature case we consider the intensity ratios of two bands of the N_2^+ first negative emission system which gives the vibrational temperature of nitrogen molecules in the ground state ($N_2 \times 1_\Sigma$). Letting numerical subscripts denote emission bands the ratio of relative intensities of two such bands may be written as[4]

$$R(T_r) = \frac{I_1}{I_0} = \frac{\Sigma_i Q_{1i}\exp - \phi_i/T_v}{\Sigma_i Q_{0i}\exp - \phi_i/T_v} \tag{19}$$

where

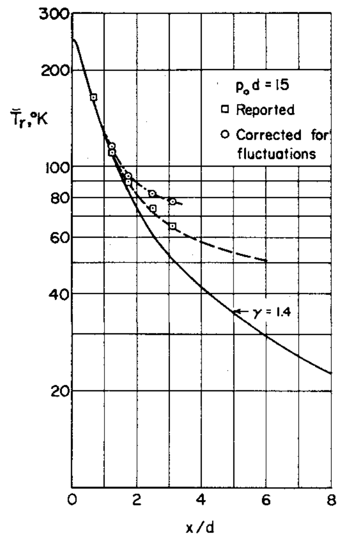
$$Q_i = q(v',v_i)$$
$$\phi_i = G(v_i)hc/k$$

Proceeding as before we arrive at

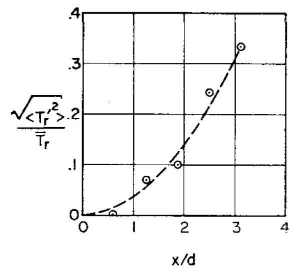
$$\overline{R(T_v)} = R(\bar{T}_v)\left[1 + \frac{\Lambda_1}{2}\frac{\langle T_v'^2\rangle}{\bar{T}_v^2}\right] \tag{20}$$

where

$$\Lambda = \frac{\tau_1 + 2\sigma_1}{\omega_1} + \frac{\tau_0 + 2\sigma_0}{\omega_0} + \frac{\sigma_0}{\omega_0}\left(\frac{\sigma_1}{\omega_1} + 2\frac{\sigma_0}{\omega_0} - \frac{\sigma_1}{\omega_0}\right)$$



a) **Reported and corrected average**



b) **Fluctuation level**

Fig. 1 Rotational temperature axial variation in an underexpanded nitrogen jet.

$$\omega = \Sigma_i Q_i \psi_i$$

$$\sigma = \Sigma_i Q_i \frac{\phi_i}{\bar{T}_v} \psi_i$$

$$\tau = \Sigma_i Q_i \frac{\phi_i}{\bar{T}_v^2} \psi_i$$

$$\psi_i = \exp - \phi_i/\bar{T}_v$$

In terms of which $R(\bar{T}_v) = \omega_1/\omega_0$. Further approximating $\overline{R(T_v)}$ by $\bar{I}_1/\bar{I}_0$ as before we obtain

$$\frac{\bar{I}_1}{\bar{I}_0} = \frac{\omega_1}{\omega_0}\left(1 + \frac{\Lambda_1}{2}\frac{\langle T_v'^2 \rangle}{\bar{T}_v}\right) \tag{21}$$

As in the case for direct emission spectroscopy we may choose a second band intensity ratio and write

$$\frac{\bar{I}_2}{\bar{I}_0} = \frac{\omega_2}{\omega_0}\left(1 + \frac{\Lambda_2}{2}\frac{\langle T_v'^2 \rangle}{\bar{T}_v}\right) \tag{22}$$

The pair of Eqs. (21) and (22) may now be solved simultaneously for $\langle T_v'^2 \rangle$ and $\bar{T}_v$ once $\bar{I}_0$, $\bar{I}_1$, and $\bar{I}_2$ are measured.

V. Experimental Results

Specific application of the direct emission results is not discussed in this paper. Attention is focused rather on selected available electron beam results both for rotational and vibrational temperatures. Rotational temperature results included the low-density highly underexpanded freejet experiments reported by Marrone,[5] the low-density wind tunnel measurements of Robben and Talbot[6] and some recent arc-heated wind tunnel results reported by Petrie.[7] Vibrational temperature measurements in an arc-heated wind tunnel reported by Petrie[7] were examined for flow fluctuation effects. All example calculations are made based on the assumption that curvature of the Boltzmann plot is due only to flow fluctuations. If, in fact, nonequilibrium curvature effects are present, additional measurements of fluctuating quantities would be required.

Rotational Temperature Application

Freejet results

The methods developed and described herein were applied to the reported results of Marrone.[5] In this case the plots of log intensity vs $K'(K' + 1)$ for the R branch were available directly (e.g., Figs. 15a, and b of the reference). First, selecting K_0' corresponding to the maximum line intensity $K_{\max}' = (T_r/2\theta_r)^{1/2} - 0.5$, where T_r was first approximated by that corresponding to a linear curve fit, a second-degree curve fit was applied using least squares. This yielded directly the coefficients a_1 and a_2 and therefore $\bar{T}_r$ and $\langle T_r'^2 \rangle/\bar{T}_r^2$ from Eq. (18). The procedure was repeated extending the range of X until overlapping and interference effects near the extremes of the R branch introduced noticeable errors. This resulted in an ensemble average $\bar{T}_r$ and $\langle T_r'^2 \rangle/\bar{T}_r^2$ deviations from which did not exceed 5%.

A typical result appears in Figs. 1a and b. Figure 1a is a plot of rotational temperature in an underexpanded nitrogen jet and corresponds to Fig. 16 of Ref. 5 for $p_0 d = 15$. $\bar{T}_r$ appears to exceed the reported T_r approaching a somewhat higher frozen value. The corresponding fluctuation levels appear in Fig. 1b, where a monatonic increase to $(\langle T_r'^2 \rangle/T_r^2)^{1/2} \approx 30\%$ is calculated. It must be recalled that for large fluctuation levels J_{10} may not necessarily be small and thus the data in Fig. 1b may reflect inaccuracy beyond 2 exit diameters.

Low-Density Wind-Tunnel Results

Correction procedures for possible fluctuation effects were the same as in the previous case utilizing, however, Robben and Talbot's $E(K', T_r)$ function [Eq. (6) of Ref. 6]. Results are summarized in Table 1. Except for the very low-temperature case corresponding to Fig. 6b of the reference the corrected temperatures are slightly higher than reported. For the low-temperature case appreciable error is expected since not enough rotational lines are available for a good statistical sample.

Arc-heated wind-tunnel results[7]

In this application of an electron beam to rotational temperature determination, the beam traverses the exit plane and the spectrometer entrance slit is aligned parallel to the beam. Thus an entire rotational temperature profile can be obtained at once. For the temperature range encountered (300–700°K) some overlapping effects from the P branch necessitated dropping a number of extreme points of the R branch. At least 15 lines were still available, however.

Results of the analysis appear as Figs. 2a and b. In Fig. 2a the reported rotational temperatures and those corrected for fluctuations are plotted vs distance from the nozzle center-

Table 1 Rotational temperature in flowing nitrogen, selected data from Ref. 6

Fig.	Reported T_r, °K	$\bar{T}_r$, °K	$(\langle T_r'^2 \rangle/T_r^2)^{1/2}$
5b	82.3	84.4	0.228
6a	34.5	35.5	0.235
6b	8.7	4.41	0.42

line for two runs. Although both runs were at nearly the same condition there is a small but noticeable displacement of the reported temperature profiles. The corrected values of rotational temperature appear to follow more closely a single curve. A possible explanation lies in fluctuating temperature plot Fig. 2b where the fluctuation level for one run was somewhat higher than the other.

It is of particular interest to note that a definite profile of rotational temperature fluctuations exist. This does not necessarily imply, however, that the nozzle boundary layer is turbulent in the classical sense. Enhancement of rotational temperature fluctuations above the freestream (centerline) value may be related to radial changes in composition.

Vibrational Temperature Application

A single example of vibrational temperature correction is included to illustrate the procedure. Petrie[8] has reported nitrogen vibrational temperature surveys across the core region of an expanded air plasma. Table 2 of Ref. 8 lists a typical vibrational temperature survey using 3 band intensity ratios. Pairs of ratios were examined systematically and pairs of equations corresponding to Eqs. (21) and (22) were solved simultaneously for $(\langle T_v'^2\rangle/\bar{T}_v^2)^{1/2}$.

An over-all average $\bar{T}_v$ was determined for each station in the profile and compared with the reported average in Fig. 3. In this case the corrected average values fall slightly below those reported.

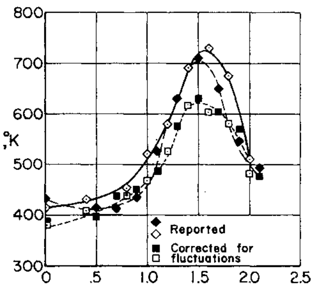
VI. Number Density Considerations

The relationship between absolute intensity of a rotational line and the N_2 number density is of the form[8]

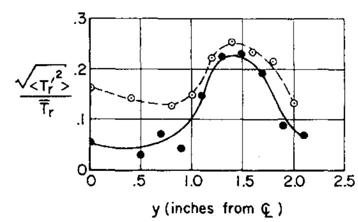
$$I_{K',K''} = S'(T_v)R(T_r)N_{N_2} = \mathrm{SRN} \quad (23)$$

If it can be assumed that temperature fluctuation effects are negligible then $I' \sim N'_{N_2}$. It is of interest to apply the foregoing methods in order to account for temperature effects and to obtain approximations to various cross-correlation quantities which arise. Thus we write

$$I_{K',K''} = F(\bar{T}_v + T_v', \bar{T}_r + T_r', \bar{N}_{N_2} + N'_{N_2}) \quad (24)$$

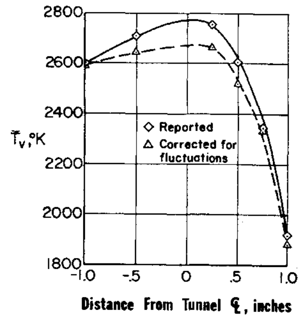


a) Reported and corrected average

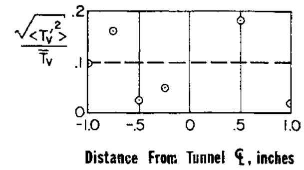


b) Fluctuation level

Fig. 2 Arc-heated nozzle flow rotational temperature.



a) Reported and corrected average



b) Fluctuation level

Fig. 3 Arc-heated nozzle flow vibrational temperature.

If Eq. (24) is expanded in a Taylor's series we have, after taking the time average and truncating to second order

$$\bar{I}_{K',K''} = S'(\bar{T}_v)R(\bar{T}_r)\bar{N}_{N_2} + \frac{1}{2}\left(\langle T_v'\rangle \frac{\partial^2}{\partial T_v^2} + \langle T_r'^2\rangle \frac{\partial^2}{\partial T_r^2} + \langle N'^2_{N_2}\rangle \frac{\partial^2}{\partial N^2_{N_2}} + 2\langle T_v'T_r'\rangle \frac{\partial}{\partial T_v}\frac{\partial}{\partial T_r} + 2\langle T_v'N'_{N_2}\rangle \frac{\partial}{\partial T_v}\frac{\partial}{\partial N_{N_2}} + 2\langle T_r'N'_{N_2}\rangle \frac{\partial}{\partial T_r}\frac{\partial}{\partial N_{N_2}}\right)_{\substack{T=\bar{T}\\ N=\bar{N}}} F \quad (25)$$

The various partial derivatives are readily evaluated, e.g.,

$$(\partial^2/\partial T_v^2)F = RN(\partial^2 S'/\partial T_r^2)$$

where

$$S' = c_1\Sigma_i[q(v',i)/Q_{\mathrm{vib}}]\exp - G(i)hc/kT_v$$

(Q_{vib} = vibrational partition function) giving

$$\left(\frac{\partial^2 S'}{\partial T_v^2}\right)_{T_v=\bar{T}_v} = \frac{c_1}{\bar{T}_v^2}\sum_i \frac{q(v',i)}{Q_{\mathrm{vib}}} \times \left[\frac{G(i)hc}{k\bar{T}_v}\left(\frac{G(i)hc}{k\bar{T}_v} - 2\right)\right]\exp - G(i)\frac{hc}{k\bar{T}_v}$$

Essentially similar steps lead to analogous expressions for the remaining partial derivatives but space prevents a complete listing. Inserting these expressions into Eq. (25) one has formally

$$I_{K',K''} = S'(\bar{T}_v)R(\bar{T}_r)N_{N_2} + C_S\langle T_v'^2\rangle/\bar{T}_v^2 + C_R\langle T_r'^2\rangle/\bar{T}_r^2 + C_{RS}\langle T_v'T_r'\rangle/\bar{T}_v\bar{T}_r + C_{NS}\langle T_v'N_{N_2}'\rangle/\bar{T}_v\bar{N}_{N_2} + C_{NR}\langle T_r'N_{N_2}'\rangle/\bar{T}_r\bar{N}_{N_2} \quad (26)$$

The coefficients are all easily derivable but are not listed here. By measuring a number of average rotational line intensities Eq. (26) can, in principle, be solved for the unknown mean square and cross-correlation terms although considerable trial and error would be required. The quantities $\langle T_r'^2\rangle/\bar{T}_r^2$ and $\langle T_v'^2\rangle\bar{T}_v^2$ can be determined independently as discussed in previous sections, and this somewhat simplifies the evaluation of Eq. (26) for the remaining cross-correlation terms. The well-known experimental difficulties in obtaining accurate absolute beam intensities owing to large fluctuations in beam current prevent meaningful numerical evaluation at present.

VII. Conclusions

The requirement to take proper account of possible effects of flow fluctuations on spectroscopic diagnostics has been demonstrated. Of course this analysis is not limited to spectroscopic techniques but may be applied whenever a nonlinear relationship can be found between a suitably normalized measured quantity and a single thermodynamic property. For example, the method should have direct application to Langmuir probe techniques.[10] Proper normalization might be accomplished by using two probes with different shapes. The net result is that the true average and mean square fluctuation components of the property in question can be obtained using conventional time average instrumentation.

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